Course Content for

3 Years B.Sc. Mathematics (Hons) Programme

(First Two Years only)

School of Mathematical Sciences teaches 5 core courses per semester, each of 75 contact hours with 4 credit points. Apart from these, there are two enrichment courses of 15 contact hours each with 1 credit point per semester. Syllabus for fifth semester is yet to be approved by Academic Council and hence is not presented here. The sixth Semester consists of projects from any of the topics listed at the end of this document.

Semester 1

Core Courses

Course 1: Elementary Calculus

Prerequisite: A reasonable knowledge on sets and functions. XI and XII standard knowledge on limit, continuity, derivative and integration.

Course Description: This is considered to be one of the foundational courses for an undergraduate mathematics student. The main concept of this course called limit of functions and limit of sequences are used to define differentiation and integration of the function respectively. The concept of differentiation is useful to understand monotonicity, concavity, convexity, linear approximation of non-linear function, existence and position of local extrema and so on. The concept of integration is useful to understand rectilinear motion and to determine length of a plane curve, area and volume of surface of revolution. The applications of average value of a function defined using definite integral will be discussed.

Course Learning Objectives: To understand the basic mathematical concepts in calculus and related theories. Understand how these concepts and ideas helped to answer some of the simple looking but very useful questions asked by not only mathematicians but also by Physicists, Scientists and many others.

Course 2: Linear Algebra

Pre-requisite:	Mathematics of Classes XI and XII (which include some basic knowledge of matrices).
Course Description:	In this course, we study system of linear equations and Gaussian elimination method; vector space, bases and its dimension; linear transformations; inner product spaces and Gram-Schmidt orthogonalization; eigenvalue problems, triangularization, diagonalization and spectral theorem for normal matrices; and canonical forms of matrices. We end the course with a glimpse of linear programming problems.
Course Learning Objectives:	Analyse and solve a system of linear equations**.** Important characteristics of matrices, such as its four fundamental subspaces, rank, determinant, eigenvalues and eigenvectors, different factorizations, etc.; How to use characteristics of a matrix to solve a linear system of equations or study properties of a linear transformation; Important concepts of vector spaces such as independence, basis, dimensions, orthogonality, etc..

Course 3: Numerical Methods

Pre-requisite:	XI and XII Standard Mathematics
Course Description:	This course emphasizes on several numerical techniques to get numerical solutions of many mathematical problems.
Course Learning Objectives:	There are many scientific and engineering problems, such as finding roots of a transcendental equation, solutions of a system of linear equations, an area under a curve, solutions of a differential equations, which often could not be treated analytically. So, numerical studies are required for these cases. After successful completion of this course the students will get a good knowledge of different numerical techniques. They can study an approximate behaviour of any mathematical model for which exact behaviour is not known.

Course 4: Algorithms and Problem Solving Techniques

Pre-requisite:	Basics of computers and school level mathematics
Course Description:	The course will train students in computer-based problem solving technique and general approach of computer languages to getting computers to solve problems.
Course Learning Objectives:	The course will give students competence in programming the computers 2) basic problems solving through a series of topics on solving various kinds of problems.

Course 5: Elementary Number theory

Pre-requisite:	Basic properties of integers, Integral exponents and powers
Course Description:	This course on Number Theory involves numbers and their properties. Composite numbers, prime numbers and operations on numbers are vital to the foundation of mathematics and its applications. Number Theory involves purest of pure mathematics but we have structured the course not only to expose numbers but also to introduce its application in a modern field like Cryptography. We first develop the concept of linear congruence and related theories which laid to the foundation of computer science. We then give a glimpse on its usefulness in introductory Cryptography. The last but not least important to mention that we focus on developing the quality of dealing with ideas rather than events.
Course Learning Objectives:	Develop the basic mathematical theories related to divisibility of integers, prime and composite numbers, and linear congruence. Understand the usefulness of this theory to introductory Cryptography.

Enrichment Courses

Apart from these, there are two short courses:

Course 6: People and personal management

This course involves discussions about how to manage your activities and time management as well as subtle points interacting with people. It also involves appreciating how people judge each other – the importance of first impressions and how to make them effective.

Course 7: Research writing and communication Skills

Communication is especially important to all of us. This may be in the form of oral communication or written communication. In order to achieve good communication, a backup work of research is also important. The course focuses on these issues.

Semester 2:

Core Courses

Course 1: Ordinary differential equations

Pre-requisite:	Elementary Calculus, Linear Algebra
Course Description:	The objective of this course is to give an idea of differential equations and how to form a differential equation. Moreover, it will introduce various methods to get solutions of differential equations exactly.
Course Learning Objectives:	After successful completion of this course the students will get a good knowledge different types of differential equations. They can study an exact behaviour of any mathematical model which is reduced to ordinary differential equations.

Course 2: Real Analysis

Pre-requisite:	Elementary Calculus
Course description:	The properties of real numbers have been the guiding force in the development of advanced and abstract mathematics which was proved to be one of the main ingredients in the development of modern scientific knowledge. Therefore, one should be pro-active to explore real number system as early as possible. The present course is a complete development of these interesting properties and their usefulness.
Course Learning Objectives:	Understand the important properties of real number systems. Understand the usefulness of these properties in advancement of mathematics.

Course 3: <u>Discrete Mathematics and Graph Theory</u>

Pre-requisite:	Mathematics at 10+2 level, Elementary Number Theory
Course Description	This course will teach students how to reason and model combinatorically. The first part of the course is based on combinatorics. After a basic introduction to sets and functions, the students will be taught various counting principles, followed by theory of generating functions and subsequently methods to obtain solutions of various recurrence relations. Some special topics like Burnside's Theorem and Polya's Enumeration Formula will also be taught. The second part of the course is based on Graph Theory. After introducing graphs, basic properties of graphs and common types of graphs like bipartite, complete, regular, connected and planar graphs will be studied. The degree sum formula, Eulerian cycles and Hamiltonian circuits will be taught. Students will be introduced to graph coloring and lastly properties of trees, spanning trees and minimum spanning trees will be studied with applications.
Course Learning Objectives:	1. Understand and apply various counting principles 2. Able to compute different generating functions 3. Solve recurrence relations 4. Study and analyze various types of graphs and understand their properties 5. Understand the concepts of Eulerian cycle, Hamiltonian circuit and graph coloring 6. Analyze and study properties of trees, spanning trees in a graph and see applications

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Course 4: Topology of Metric Spaces

Pre-requisite:	Elementary Calculus and Real Analysis
Course Description:	Gives a streamlined development of a course in Metric space topology, emphasizing the most useful concepts, familiarizing with concrete examples of spaces, and geometric ideas to encourage geometric thinking.
Course Learning Objectives:	(1) To introduce the primitive concepts of open sets and introduce other concepts via that concept. (2) To discuss geometric motivations of concepts and results and lots of concrete and geometric examples and pictures.

Course 5: Elementary Probability

Pre-requisite:	Calculus, Discrete Mathematics
Course description:	Any realistic model of real world phenomenon must take into account the possibility of randomness. That is, more often than not, the quantities we are interested in will not be predictable in advance but, rather, will exhibit an inherent variation that should be taken into account by the model. This is usually accomplished by allowing the model to be probabilistic in nature. Such a model is, naturally enough, referred to as a probability model. In order to master both the "model building'' and the subsequent analysis of these models, one must have significant knowledge of basic probability theory. The present course is designed to give students a solid understanding of elementary probability.
Course Learning Objectives:	Understand the basic theory of probability and the meaning of probabilistic model. Understand how to analyse probabilistic model using the basic theory of probability.

Enrichment courses

Apart from these, there are two short courses:

Course 6: History of Indian mathematics

Mathematics began with numbers designed for keeping count of things. From there it went on to geometry and algebra and soon the field became a huge

collection of ideas that evolved over several millennia. In this course, we will discuss how mathematics evolved.

Course 7: Environmental Sciences

Environment is under stress to an unprecedented scale. It is therefore important for young students to become aware of the complexity of the issues related to environment

Semester 3:

Core Courses

Course 1: Complex Analysis

Pre-requisite:	Elementary Calculus, Real Analysis
Course Description:	We begin the course with properties of complex numbers. We study the differentiability and contour integration of complex valued functions in one complex variable. We further study zeros and poles of complex valued functions in one complex variable to evaluate contour integrations. We see applications of contour integration to evaluate definite real integrals.
Course Learning Objectives:	Properties of Complex Numbers, Differentiability of Complex Valued Functions in One Complex Variable, Contour Integration of Complex Valued Functions in One Complex Variable, Evaluation of Definite Real Integrals

Course 2: Multivariate Calculus

Pre-requisite:	Elementary Calculus and Real Analysis
Course Description:	We first see different ways of representing functions of several variables. We introduce partial derivatives and use it to get various local information about the function such as tangent planes and directional derivatives. We will develop various techniques such as second derivative test and Lagrange multiplier methods to find local and global maxima and minima of a multivariable function. Then we discuss some of the most important theorems including Boundedness Theorem, Extreme Value Theorem, Implicit

	Function Theorem, Inverse Function Theorem, and their applications.
Course Learning Objectives:	Many quantities in various scientific fields depend on more than one variable. In this course we understand how the concept of limit, continuity, differentiability and integrability is defined for such quantities; the mathematical techniques required to handle real world problems involving such quantities.

Course 3: <u>Theory of Optimization</u>

Pre-requisite:	Linear Algebra, Elementary Calculus, Discrete Mathematics
Course Description	In this course, we study optimization problems, namely, Linear Programming Problem, Game Theory, Convex Programming Problem and Nonlinear Programming Problem. In Linear Programming Problem, we study the Duality Theory, Transportation Problem, Network Flows, etc. In Game Theory, we study Two-Persons Zero Sum Game and Rectangular Game. In Convex Programming Problem, we study Optimization in Standard Form, Local and Global Optima, and an Optimality Criterion. In Nonlinear Programming Problem, we study Wolfe's Method and Beale's Method for quadratic programming problem, and Karush-Kuhn-Tucker Theory.
Course Learning Objectives	Analyse and solve Linear Programming Problems, Transportation Problems in Real Life, Network Flows, Two-Persons Zero Sum Game and Rectangular Game, Analyse and solve Convex Programming Problems, Analyse and solve Nonlinear Programming Problems

Course 4: <u>Partial Differential Equations</u>

Pre-requisite:	Elementary Calculus, Linear Algebra, Ordinary Differential Equation
Course Description:	The objective of this course is to give an idea of differential equations and how to form a differential equation. Moreover, it will introduce various methods to get solutions of differential equations exactly.

Course Learning Objectives :	After successful completion of this course, the students will get good knowledge different types of differential equations; they can study an exact behaviour of any mathematical model which is reduced to ordinary differential equations.

Course 5: <u>Abstract Algebra</u>

Pre-requisite:	A reasonable knowledge on sets, relations and functions. and Basic knowledge in elementary number theory.
Course Description:	This is considered to be one of the foundational courses for an undergraduate mathematics student. The main concept of this course is to make students familiar to basic objects in abstract algebra like groups and rings. On one hand, while they will be introduced to the axioms of group theory and ring theory, on the other hand they will be taught to find appropriate models of the theory in various structures through plenty of examples. They will be introduced to some celebrated theorems and results in these fields and also will be taught to deduce interesting consequences of those results.
Course Learning Objectives:	Understand the basic mathematical concepts in group theory and ring theory. Understand how these concepts and ideas helped to answer some of the simple looking but very useful questions asked by not only mathematicians but also by Physicists, Scientists and many others. Ability to write correct mathematical proofs.

<u>Enrichment courses:</u>

Apart from these, there are two short courses:

Course 6: History of Mathematics in India.

Mathematics in India has a unique path and exploring the same provides a good insight into the very nature of Mathematics.

Course 7: Literature.

Literature written by Mathematicians such as Alice on Wonderland provide a fascinating insight into mathematics, literature and perspective of Mathematicians to the world

Semester 4:

Core Courses

Course 1: Vector Analysis

Prerequisite:	Elementary Calculus, Linear Algebra, Multivariate Calculus
Course Description:	Students will learn to solve problems in three-dimensional space by utilizing vectors and vector-algebraic concepts. This includes representation in Cartesian, cylindrical and spherical coordinates. We first see different ways of representing functions of several variables. Then we discuss line integrals, multiple integrals, and surface integrals, with applications to vector analysis.
Course Learning Objectives:	Notion of a definite integral from a one-dimensional to an n-dimensional space, and be able to describe and evaluate double and triple integrals in Cartesian and curvilinear coordinates. To work with vector-valued functions of several variables (i.e., vector fields) and be able to compute line and surface integrals. To use the theorems of Green, Stokes, and Gauss to solve classical physics problems.

Course 2: Functional Analysis

Prerequisite:	Linear Algebra, Real Analysis
Course Description:	We begin the course with Linear Spaces and Linear Maps. We study Metric Spaces and Continuous Functions. We study Banach Spaces with an emphasis on L^p Spaces. We study Bounded Linear Maps on Banach Spaces and the Spectrum of Bounded Operators on Banach Spaces. We further study Hilbert Spaces and Bounded Operators on Hilbert Spaces.
Course Learning Objectives:	To understand Banach Spaces, Bounded Operators and their Spectrum on Banach Spaces, Hilbert Spaces, Bounded Operators on Hilbert Spaces

Course 3: Measure Theory

Prerequisite:	Real Analysis, Elementary Probability
Course Description:	Measure Theory formalises and generalises the notion of integration. It is fundamental to many areas of mathematics and probability and has applications in other fields such as physics and economics. This is a course without which the knowledge of probability theory is impossible to apply in modern day real life problems, for instance, stock market. A lot of seemingly elementary applications of probability theory will be better understood if viewed through the eyes of somebody who studied measure theory.
Course Learning Objectives:	After completing this subject, Students will understand the fundamentals of Lebesgue theory, they will be acquainted with the proofs of the fundamental theorems underlying the theory of integration, they will get a glimpse on how abstract theory of measure and integration is developed as generalized Lebesgue theory, they will learn about one of the most important function spaces namely, L^p -spaces.

Course 4: Topology and Geometry

Prerequisite:	Linear Algebra, Multivariable Calculus, Real Analysis, Ordinary Differential Equations, Topology of Metric Spaces
Course Description:	In this course, differential geometry will be taught. Firstly, the theory of curves will be introduced through the concepts of coordinates, arclength and curvature. Secondly, surfaces will b e studied. The Gaussian curvature, an analog of curvature of curves for surfaces, will be introduced. Students will also be taught the concepts of tangent plane and unit normal. It will be shown that Gaussian curvature is an intrinsic feature of a surface. The concept of vector fields and parallel transport will be introduced. Students will be taught geodesics. They will be made familiar with important results like the Hopf-Rinow Theorem and Hopf's Umlaufsatz. Finally, Gauss-Bonnet theorem will be proved and some applications will be given. After the part in differential geometry has been completed, students will be taught some basic topics in algebraic topology like homotopic maps and the fundamental group.

	Fundamental group of the real circle will be computed. The students will be introduced to the Van Kampen's Theorem (proof to be done if time permits) and some applications will be given.
Course Learning Objectives:	The students will be able to Analyse and study curves and surfaces through measures of curvature, arc-length etc.; To be able to compute tangent plane and unit normal; Understand the concepts of coordinate charts, differentiable maps between surfaces; Analyse and study different curvatures of surfaces; Understand the concept of geodesics; Understand the concept of homotopy and the fundamental group; Study applications of the results learned.

Course 5: <u>Statistics and Machine Learning</u>

Pre-requisite:	Probability, Basic programming
Objectives:	By the end of the course, students should be able to: Develop an appreciation for what is involved in learning models from data. Understand a wide variety of statistical learning algorithms and evaluation of models generated from data. Apply the algorithms to a real-world problem. Get a glimpse of deep learning techniques
Course Learning Objectives:	To understand and implement statistical machine learning algorithms and get an exposure to deep learning techniques.

<u>Enrichment courses</u>

Apart from this, there are two short courses:

Course 6: Microeconomics. Micro economics heavily involves mathematical ideas and concepts. The course explores basic concepts of micro economics and its mathematical ideas.

Course 7: Presentation Techniques. These techniques are crucial to students as they move out of their comfort zone of class room and move into real world where they need to present their work and plan in a manner which is effective in front of unknown audiences. The course highlights these issues.

Semester 5: Courses are under approval

Semester 6: Students are expected to do project on any of the topic listed below:

(1) Fine and Performing Arts
(2) Social Organisation
(3) Social Dynamics
(4) Statistical Studies
(5) Business Analytics
(6) Management
(7) Computer Science
(8) Data Science
(9) Simulations and Animations
(10) Engineering
(11) Industrial Mathematics
(12) Physical Sciences
(13) Biomathematics
(14) Earth Sciences
(15) Mathematics
(16) Education
(17) Any Other Topic of Human Learning
(18) Online Course with Lesser Emphasis on Project Activity