SVKM's NMIMS

School of Mathematical Sciences

Programme: B.Sc. Mathematics (Honours)

Academic Batch: 2020-23

Date: June 16, 2020

Full Marks: 60

Time: 14:30 – 16:00

Duration: 90 minutes

No. of Pages: 4

Entrance Examination

There are 30 *questions of multiple choice type. For each question, only* **one** *option is correct. Each question carries* 2 **marks**. *There is* **negative** *marking.* 2 **marks** *will be awarded for each* **correct** *answer,* -1 *for each* **incorrect** *answer and* 0 *if the question is* **not attempted**.

(1) If m men can do a job in d days, then the number of days in which $m+r$ men can do the job is

(A)$d+r$; (B)$\frac{d}{m}(m+r)$; (C)$\frac{d}{m+r}$; (D)$\frac{md}{m+r}$.

(2) The digit in the unit position of the integer

$$1! + 2! + 3! + \cdots + 99!$$

is

(A)3; (B)0; (C)1; (D)7.

(3) Four statements are given regarding elements and subsets of the set $\{1, 2, \{1, 2, 3\}\}$. Which is the correct statement ?

(A) $\{1, 2\} \in \{1, 2, \{1, 2, 3\}\}$; (B) $\{1, 2\} \subseteq \{1, 2, \{1, 2, 3\}\}$;

(C) $\{1, 2, 3\} \subseteq \{1, 2, \{1, 2, 3\}\}$; (D) $3 \in \{1, 2, \{1, 2, 3\}\}$.

(4) For which value of a the system of equations

$$2x + 3y = 10$$

$$5x + ay = 25$$

has infinitely many solutions?

(A)$\frac{15}{2}$; (B)$\frac{2}{15}$; (C)$\frac{15}{4}$; (D)$\frac{4}{15}$.

(5) The solutions of $2\tan^2 x + \sec^2 x = 2$ in the interval $[0, \pi)$ are

(A)$\frac{\pi}{6}, -\frac{\pi}{6}$; (B)$\frac{\pi}{3}, \frac{2\pi}{3}$; (C)$\frac{\pi}{6}, \frac{5\pi}{6}$; (D)$-\frac{\pi}{6}, \frac{5\pi}{6}$.

(6) If $y = (\cos^{-1} x)^2$, then the value of $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$ is

(A)-1; (B)-2; (C)1; (D)2.

(7) Let

$$g(x) = \int_{-10}^{x} tf'(t)dt \quad \text{for } x \geq -10,$$

where f is an increasing function. Then

(A) $g(x)$ is an increasing function of x;

(B) $g(x)$ is a decreasing function of x;

(C) $g(x)$ is increasing for $x > 0$ and decreasing for $-10 < x < 0$;

(D) none of the foregoing conclusions is true.

(8) If the straight line $x = y$ is tangent to the parabola $y = x^2 + bx + c$ at the point $(1, 1)$, then

(A) $b = -1$, $c = 1$; (B) $b = 1$, $c = -1$; (C) $b = 0$, $c = -1$; (D) $b = -1$ and c arbitrary.

(9) The curve in the complex plane given by the equation $Re(\frac{1}{z}) = \frac{1}{4}$ is a

(A) vertical straight line at a distance of 4 from the imaginary axis;

(B) circle with radius unity;

(C) circle with radius 2;

(D) straight line not passing through the origin.

(10) If z is a non zero complex number such that $\frac{z}{1+z}$ is purely imaginary, then z

(A) can be neither real nor purely imaginary;

(B) is real;

(C) is purely imaginary;

(D) satisfies none of the above properties.

(11) If x, y, z are arbitrary positive real numbers satisfying the equation

$$xy + yz + zx = 9,$$

then the maximum possible value of the product xyz is

(A)$\sqrt{27}$; (B)$4\sqrt{3}$; (C)5; (D)$3\sqrt{6}$.

(12) Let i denote the complex number $\sqrt{-1}$. Then the roots of the quadratic equation $x^2 + ix + 2 = 0$ are

(A)$i, -i$; (B)$i, -2i$; (C)$-i, 2i$; (D)$i, 2i$.

(13) If $2 + 3 + 4 + \cdots + n = 54$, then n is

(A)8; (B)9; (C)10; (D)11.

(14) If ${}^{11}C_4 = {}^{n}C_{n-4}$, then n is

(A)4; (B)7; (C)8; (D)11.

(15) $\int_0^{\frac{\pi}{6}} (2\cos^2\theta - 3\sin^2\theta)d\theta =$

(A)$\frac{5\sqrt{3}}{2} - \frac{\pi}{12}$; (B)$\frac{5\sqrt{3}}{4} - \frac{\pi}{12}$;

(C) $\frac{5\sqrt{3}}{8} - \frac{\pi}{12}$; (D)$\frac{5\sqrt{3}}{16} - \frac{\pi}{12}$.

(16) If you have a set of 10 natural numbers, then it has a subset such that sum of the elements in the subset is divisible by 10.

(A) True (B) False.

(17) The number of positive roots of the equation $x^4 + 3x^2 + 2x - 1 = 0$ is

(A)0; (B)1; (C)2; (D)3.

(18) $\frac{1}{5^2} + \frac{2}{5^4} + \frac{3}{5^6} + \ldots\infty =$

(A)$\frac{25}{576}$; (B)$\frac{1}{25}$; (C)$\frac{5}{24}$; (D)$\frac{1}{24}$.

(19) Let S be the set of all numbers of the form $4^n - 3n - 1$, where $n = 1, 2, 3 \ldots$. Let T be the set of all numbers of the form $9(n-1)$, where $n = 1, 2, 3, \ldots$. Then which is the correct statement ?

(A)$S \subseteq T$; (B) $T \subseteq S$;

(C)$S = T$; (D)$S \nsubseteq T$ and $T \nsubseteq S$.

(20) The number of four digit numbers greater than 5000 that can be formed out of the digits 3, 4, 5, 6 and 7, no digit being repeated is

(A)52; (B)60; (C)69; (D)72.

(21) Suppose $a+b+c$ and $a-b+c$ are positive and $c<0$. Then the equation $ax^2+bx+c=0$

(A) has exactly one root lying between -1 and 1;

(B) has both the roots lying between -1 and 1;

(C) has no root lying between -1 and 1;

(D) nothing definite can be said about the roots without knowing the values of a, b and c.

(22) Which of the following is a surjective function ?

(A) $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=x^3$; (B) $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=x^2$;

(C) $f:(\frac{1}{3},1)\to[\frac{1}{9},1]$ given by $f(x)=x^2$; (D) None of the above.

(23) Graph of a polynomial intersects the y-axis at 1 point and the x-axis at 3 points. Then the number of real roots the polynomial has is

(A)1; (B)2; (C)3; (D)4.

(24) $\lim_{x\to 2}\ \dfrac{x^2+x-6}{x^2+3x-10}=$

(A)0; (B)1; (C)$\dfrac{3}{7}$; (D)$\dfrac{5}{7}$.

(25) Which of the following is the area under the curve $y=\cos(x)$ above the interval $[-\pi,\pi]$?

(A)0; (B)1; (C)2; (D)2π.

(26) If 2^n-1 is prime number, then n is a prime number.

(A) True (B) False.

(27) If 3 fair dice are cast, the probability that the sum is 11 is $\dfrac{1}{8}$.

(A) True (B) False.

(28) There exist 4 points on the plane such that the line joining any 2 points is perpendicular to the line joining the other 2 points.

(A) True (B) False.

(29) Let A be the 2×2 matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Then A^{102} is

(A)A; (B)$-A$; (C)I_2; (D)$-I_2$,

where I_2 denotes the 2×2 identity matrix.

(30) The polynomial function $f(x)=2x^3-3x^2-12x+6$ has a maxima at $x=-1$.

(A) True (B) False.

PAPER ENDS